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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

VOL. 7 BATON ROUGE, LA., OCTOBER, 1932 No. 1

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ALFRED COLLEGE

Alfred, N. Y.

March 20, 1932.

My dear Professor Sanders:

I want to thank you for the last two copies of the Mathematics News Letter, a very much worth while publication. Lacking the pretentiousness of some of our other journals, your little "News Letter" does not need to make any apologies for the quality of its content. I congratulate you and the Editorial Board.

Some time I should like to comment on your last editorial,—"Are We Shamming?" My own observations have been that college teachers sin more often at the other extreme.

Under separate cover I am sending you the results of my investigation of college teaching. When you will have read it, be good enough to send me your reactions to it and your general criticism of it. The last issue of *The American Mathematical Monthly* contained a brief review of my study.

Wishing you continued success in your noble efforts, I am,

Yours, in a common cause,

JOSEPH SEIDLIN.

UNIVERSITY OF WISCONSIN

Madison, Wis.

February 22, 1932.

Professor S. T. Sanders,
Department of Mathematics,
Louisiana State University,
Baton Rouge, Louisiana.

Dear Professor Sanders:

Enclosed please find a check for $2.50, one dollar of this is for my subscription to "Mathematics News Letter" and $1.50 is for ten copies of the News Letter, volume 6, number 3, November-December, 1931, in which you have an editorial entitled "*The Angle-Trisection Chimera Once More*". If it were possible for you to send me reprints of this editorial rather than full copies of the News Letter, I would like to have you do so up to the number that $1.50 would pay for. I am going to use these copies to send out to people who write in about trisecting an angle.

May I at this time express my appreciation of the splendid work that you and your colleagues are doing in furthering the interests of mathematics.

Sincerely,

Signed:() MARK H. INGRAHAM.

GEOMETRY IN THE LOUISIANA HIGH SCHOOLS

In the September issue of "Louisiana Schools", journal of the Louisiana Teachers' Association, appears an elaborate recommendation offered by State High School Supervisor Trudeau to Superintendent T. H. Harris, the recommendation suggesting a radical change in the high school curricula of the State. Six high school curricula are outlined in detail by Mr. Trudeau, namely, "College - Preparatory", curriculum, "Home Economics" curriculum, "Agricultural" curriculum, "Commercial" curriculum, "Industrial Arts" curriculum and "General" curriculum. Analysis of each of these curricula shows that plane geometry is a required unit in but one of them. That one is the College - Preparatory curriculum. In the tenth grade of the Home Economics curriculum geometry is made optional. The student is permitted to choose three units out of five offered, namely, Home Economics, Foreign Language, Social Study, Plane Geometry, Chemistry or Physics. Of the ten ways in which he may choose three courses from this group there are four that allow him to leave out the geometry. In the Agricultural curriculum the student has three ways of avoiding it in six choices. In the Commercial curriculum, he must elect two units from the five offered, one of the five being geometry. Thus of the ten ways of choosing his two units, six do not include the geometry. In the tenth grade of Industrial Arts Curriculum the ratio of choice is the same as in the Home Economics curriculum, namely, of ten choices of unit combinations, four leave out the geometry. This, also is the General curriculum ratio.

As one of many whose duty it is from year to year to teach and to supervise the teaching of college mathematics we may be pardoned for some frank comment upon this new status of geometry, a status which, we are given to understand, is already operative in the Louisiana schools.

It is our conviction that the policy is questionable which permits the most ancient of the school disciplines, Euclid's geometry, to be placed at the mercy of an uninformed and largely an unreflecting choice of sixteen-year old boys and girls. Certainly it is an extension of the principle of option in school subjects, the value of which must be in grave doubt to say the least. Many of these young people if left to their own option might have chosen to avoid school altogether. But such a choice is forbidden by the State. Yet in all of our secondary programs except one we are permitting them to choose to remain wholly ignorant of a field of study which has been a classic requirement in schools of Europe and of England for more than a thousand years. A far more rational use of the option principle would be to demand that every student before he makes a choice among several fundamental branches of learning should acquire a beginning knowledge of them all. When he shall have attained some knowledge of what the subject matter is about, then, not until then, will he be able to make a somewhat intelligent answer to the questions: To what kind of subject matter is my mind best adapted? What subject do I like best? In which field is my interest greatest? The five books of Euclid—usually called plane geometry—constitute only the initial chapter of a division of mathematics, known as one of the three major divisions of the science. When a boy or girl is free to choose to remain ignorant of even this initial chapter of it, such a choice has no significance as a choice. In no sense whatever is it one based upon "taste and aptitude." The existence or non-existence of "aptitude" can only be discovered by giving the subject a trial.

In view of this, is it fair to the student to make him the unwitting victim of a system of options by allowing him to go through high school with his powers in geometry totally untested?

The same lack of informed judgment that is apt to lead the high school student into a mistake in choosing his courses

is just as often exhibited when he comes to decide, at the age of sixteen or seventeen, whether, or not, he will go to college or enter a vocation, upon leaving high school. We very seriously question the wisdom of having the secondary courses so organized that the high school graduate who did not choose the college-preparatory curriculum, must enter college, gravely handicapped. Many have entered thus handicapped because their decision not to enter college was revised after leaving high school—but revised too late. Plainly, if the freshman year at college reveals to such a student a sufficient degree of mathematical ability to make him desire to major in that field, the handicap must then be disclosed. He chose no geometry in high school and cannot repair the omission by taking it in college since it is a secondary, not a college subject matter.

Such a grave gap in the correlation of college and high school mathematical curricula may, fortunately, be remedied with but little trouble. Let the State Department of Education restore the so-called traditional subject Geometry to its original firm place in all high school curricula, thus re-assigning to all of them a COLLEGE PREPARATORY character.—S. T. S.

INTRODUCTION AND PRESENTATION OF THE FORMULA AS THE FIRST UNIT IN NINTH MATHEMATICS

By MRS. MARTIN S. RILEY
State Teachers' College
Hattiesburg, Miss.

A problem which for many years has seemed to confront pupils, teachers, and interested parents is the gap which often exists between eighth and ninth mathematics, or in many schools, between eighth arithmetic and the first year of algebra.

Some attempts at a solution have been: (1) beginning ninth mathematics with a review of the fundamental principles of eighth (This only delays the main problem.); (2) separating mathematics into junior and senior high school courses, which is possible usually only when the system is organized on the 6-3-3 plan (The purpose of

the first part of this series is to give a pupil completing the junior course all the mathematics that a person of ninth grade ability would need in any occupation; consequently the three years, seven, eight, and nine, would have given a generalized course including what we term, arithmetic, algebra, plane and solid geometry, trigonometry, and business arithmetic.); (3) beginning ninth mathematics in a way which will bridge the gap—that is, connecting the fundamentals of algebra directly with some emphasis of eighth mathematics, whether it has been what we might term pure arithmetic or a generalized course.

This article attempts to give an introduction and presentation of the formula as the first unit in such way that for the majority of pupils, the gap, whether real or imaginary, will be bridged and a continuity of mathematics study established. Space does not permit the listing of aims, habits, attitudes, methods of work, concepts, and abilities for this unit, but the presentation is based on a very carefully prepared list. This preparation should be made by every teacher before a unit of work is begun.

Introduction or preview.

This may be given by means of an inventory test and discussion which the teacher leads:

"In the study of algebra we shall find many likenesses to arithmetic; all that was true there is true in algebra, but we shall find algebra much broader and dealing with new tools. One important tool which we found in arithmetic we shall find as a basis for the most of algebra—that is, the formula in its basic meaning—the equation. During our first unit we shall explore the world about us to find uses of the formulas dealing especially with plane and solid figures and simple scientific problems.

"Will someone give us a formula? (If no one can give a formula, a rule for some simple area will be requested. Should this fail, the rule and formula for the area of a rectangle will be developed. The comparison of the rule and formula will be made through class discussions.) Are there many objects or parts of the classroom for which it was necessary for someone to use formulas in some computations? (Time should be given for discussion on this.) Are there any illustrations of formula use in the building or elsewhere? (This should lead to an assignment whose completion involves several lessons—that is,

the collection of formulas and the listing of objects and conditions requiring their use.) To determine which formulas you know and how much we shall not need to study I have prepared an objective test which you will want to answer to the very best of your ability. (Give here an exploration test of about twenty minutes in length. This should include the naming of the simple figures, the illustration of designated ones, identifying formulas, stating formulas, and determining the formula necessary under given conditions.)"

Presentation.

Several days will be spent in the continuing of the quest which was begun in the introduction. Each day what the pupils have found will be discussed, problems solved thereby, and illustrated objects identified. The formulas as found will be grouped according to their uses with plane or solid figures and other sciences. Under each grouping a careful classification will be made and for each formula the name and illustration of the figure (if there is one) will be given. After all voluntary contributions have been made, assignment will be made on areas, volumes, surfaces, and perimeters of the plane and solid figures not already listed. When this work is completed each pupil will be required to make a correctly classified list to be checked by the teacher and to be filed in his notebook for reference. Since this is the pupil's main reference for several lessons, each individual's list must be complete and accurate in every respect. Drill will follow; this should involve the identifying of geometric figures and their parts (using the complete set of planes and solids), spelling their names, giving formulas for areas, volumes, perimeters, and surfaces, evaluating in the original form, determining what is required from word problems and the solution thereof. This will be accomplished by means of problems submitted by the pupils, the making of models by volunteers, assignments, class drill, workbooks, and short, frequent tests. Records will be kept carefully of all work done by each pupil. Scientific formulas given in the text used and simple ones submitted by the pupils will be discussed, and drill in evaluation only will be given except in the study of the two temperature formulas, Fahrenheit and Centigrade. (These will be developed and studied completely, partly because of the correlation with General Science which most of the pupils will study during the session of the formula study.)

After the careful study of the formulas in the above discussion the evaluation of general algebraic statements will be presented in the following order:

Given: the value of a, b, and c,

Evaluate: 2a

$2a+3b$, and $3a+4b-2c$, etc.

$a/3$, $\frac{2a}{4}$, $\frac{2a}{4}+\frac{3b}{6}$, $\frac{4b}{3}-\frac{3c}{5}$, etc.

$(a+b)^2$, a^2+b^2, etc.

The next division will be the use of symbolic language for shortening word statements—that is, the making of formulas, and the interpretation of symbolic language in the following order:

1. Involving one relation,—one number is 8 greater than another; one number is 8 less than another; one number is three times another number; a number divided by 4. The terms—sum, difference, product, and quotient—will be stressed.
2. Involving two relations,—two times a number increased by 8; two times a number decreased by $\frac{1}{3}$ itself, etc.

Daily drill, tests, and workbooks will be used.

Following the work with symbolic language will be equation formation developed on this plan:

Problem: The length of a rectangle is $2\frac{1}{2}$ times the width and the perimeter is 98 feet. What are its dimensions?

Solution: Let x = the number of feet in width

Then $2\frac{1}{2}x$ = the number of feet in length

$2(x)+2(2\frac{1}{2}x)$ = the number of feet in perimeter

98 = the number of feet in perimeter

$$2(x)+2(2\tfrac{1}{2}x)=98$$

Following formula or equation formation will be the solution of equations. This will be introduced by developing the axioms by the laboratory method,—that is, using a pair of balances and experimenting to determine results when equals are added to equals, equals are subtracted from equals, etc. After axioms are derived, the equations formed previously (involving one and two relations) will be taken for solution in this order:

1. Using division axiom, such as $2x = 18$.
2. Using multiplication axiom, such as $x/2 = 5$.
3. Using subtraction axiom, such as $x + 8 = 15$.
4. Using addition axiom, such as $x - 10 = 5$.

Stress will be placed upon keeping the equation balanced or on performing the same operation on both sides—changing the form but not the value.

Included in the equation solution by use of one axiom should be the evaluation and solution of formulas of the same type:— *e. g.*, given $A = bh$, find h when $A = 30$ and $b = 6$. After correct substitution is made, the problem becomes the solution of the equation, $30 = 6h$. (Use this method of solution rather than the three formulas; $A = bh$, $b = A/h$, $h = A/b$.) The next work should be the solution for a particular letter:

Given:	$A = bh$	
Required:	to find b in terms of A and h	
Solution:	$A = bh$	Given
	$\frac{A}{h} = \frac{bh}{h}$	If equals are divided by equals, the quotients are equal.
	$\frac{A}{h} = b$	Dividing both numerator and denominator of a fraction by the same quantity does not change its value.

After equations (including formulas) are solved by one axiom, then those involving more than two steps will be studied. Too lengthy

equations should not be attacked until more tools which will be introduced in other operations have been acquired.

The final phase of this study will be the graphing of equations or formulas—*e. g.*, a graph of $C=21g$ (cost of gasoline at 21c. per gallon). Its complete construction and use will be taught and the advantages of its use stressed After the graphing of the formula the next presentation will be the use of the graph to solve problems the equations for which would be difficult to make and solve at that time—for example,

> A truck leaves a town and travels at the rate of 30 miles an an hour. Three hours later a car leaves that town and travels over the same road at the rate of 40 miles an hour. How soon after the car leaves will the truck be overtaken by it?

(The construction of each graph on the same axis is simple; the intersection of the graphs when properly interpreted gives the solution.) Using the graph as the last phase of the formula and equation unit gives a good foundation for the signed numbers and for further work with graphs and equations.

ON TEACHING MATHEMATICS FOR DISCIPLINE

By W. PAUL WEBBER
Louisiana State University

There is quite a prevalent idea that the study of mathematics affords a discipline of a valuable kind. It may not be clear to all as to just what is meant by discipline in this connection. We may get an idea of its meaning from various walks of life.

(1) What does discipline mean to the military man?

A man who has acquired a set of habits and attitudes that enable him to react correctly and efficiently to any command or military problem in the field is said to be disciplined in the military sense.

It is a matter of common observation that this training enables the man to react correctly and efficiently in some situations not military in character. It is not the purpose of this paper to discuss how this comes about.

(2) What does discipline mean in the family?

A member of a family is disciplined if he has acquired such habits and attitudes as lead him to cooperate in the solution of the family problems and to so control his actions as not to raise his individual preferences above the family welfare.

Again it is a matter of common observation that a member of such a family correctly and efficiently reacts to many problems outside the family circle in the larger social environment.

(3) What does discipline mean in the school room?

A school is said to be well disciplined if the pupils acquire attitudes and habits that restrain them from obstructing the purpose for which the school is conducted and that lead them to cooperate in making the realization of that purpose as full as possible.

Here again it is a matter of observation that the pupils of such a school show themselves prepared to cooperate in larger social activities with credit to themselves and their community.

We of course admit that a small number of individuals in every group successfully resist the discipline afforded and do not obtain all the benefits that may be available.

(4) What do we mean by discipline in the intellectual field?

We should expect that higher education should carry the habit forming process into the realm of the more purely intellectual activities.

In this realm the ability to efficiently use the correct forms of thinking and to recognize deviations from these forms on the part of others are of the highest importance. All this implies the existence of, and the desirability of attaining, a status that may be termed an intellectual life as supplementary to the ordinary sensuous life. Not all are endowed by nature and childhood back ground to travel far into field of intellectual life. Some can attain the intellectual life with little or no help from the formal school instruction. Proper school training should be a great aid to the majority of aspirants for this accomplishment.

The National Committee on Secondary Mathematics laid down the principle: "The primary purposes of teaching mathematics should be to develop those powers of understanding and of analysing relations

of quantity and space which are necessary to an insight into and a control over our environ,ent and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual". Now any one must admit this is a large order and a broad claim. Have teachers of mathematics and text book writers lived up to this doctrine? I would say that up to the word "environment" they have made a fair showing. From that word to the end of the statement I think little has been accomplished or attempted. Some teachers have published texts that purport to be intended especially for liberal arts students. But to me the most artistic thing about these books is their mechanical perfection in the printing and binding. There is little or no specific material to especially appeal to or serve the needs of the non-mathematical college student. Do these men deceive themselves or are they unable to write what they think? The material is still almost entirely technical mathematics with no very unusual problem material to carry over the training into other fields of thought.

There has been progress in teaching mathematics as such and this is to be commended, but I do not think the full implication of the claim laid down by the committee has appeared to teachers. There is no certainty that the committee fully realized this.

If the assumption of a considerable number of mathematicians, namely, mathematics is identical with or includes logic as one of its parts, is valid, there is much to be done by teachers of mathematics to make it the educational instrument that mathematics teachers have claimed it to be.

I hold that it has been settled that the modes of thinking in life and in mathematics are substantially the same, though this fact has not been made sufficiently clear either in books or in teaching. Professor C. N. Moore touched on this in a paper in *Education*, December, 1918, by use of an example from political history. Others have made similar contributions in different situations.

If mathematics is logic and the pattern of logical procedure, then why must we so exclusively limit our teaching of thinking to number and space? These are valuable and fundamental in certain fields, but we should go further and show by actual examples in the elementary books that thinking in other fields is done on the same pattern as in mathematics. It is one purpose of this paper to give a few suggestive examples of material for extending mathematical thinking

into other fields of intellectual life. Some exercises taken from texts on traditional logic are here appended, and the question is seriously asked whether simple exercises of this character should not be found in high school and junior college texts on mathematics. It would seem that they might mix with exercises in geometry quite easily. It is expected that these exercises are to be analysed and put in typical syllogistic form and the truth or falsity of each determined and faulty arguments crticised.

1. Men are mortal beings. Kings are men. Therefore kings must die.
2. The right should be enforced by law. The exercise of suffrage is a right and therefore should be enforced by law.
3. Rational beings are accountable for their actions. Brutes are not rational beings. Therefore——
4. Mathematical study improves the reasoning powers. Logic is not a mathematical study. Therefore the study of logic will not improve the reasoning powers. What is wrong here, if anything?
5. The end of any thing is its perfection. The end of life is death. Therefore—
6. Whoever intentionally kills another human being should suffer death. A soldier kills a man on the enemy lines. Therefore—
7. Books are a source of amusement. A table of cosines is a book. Therefore—
8. Who is most hungry eats most. Who eats least is most hungry. Therefore—
9. What we eat grows in the field. We eat loaves of bread. Therefore—
10. Stonemasons are benefitted by the mason's union; bricklayers are benefitted by the bricklayers union, etc., in fact every trade is benefitted by its union. Therefore, if all workmen had unions all workmen would be benefitted.
11. Improbable events happen almost every day, but what happens every day is most probable. Therefore improbable events are most probable.

It would be possible to make use of problems involving longer trains of inference. Such would approach the more complex mathematical problems in difficulty.

It is not suggested that the above exercises are the best that could be found for the purpose. The question is raised as to whether questions of the same general kind as these might profitably be mixed in with purely mathematical exercises of the traditional type. The writer would be glad to have some opinions from others and some sample exercises, also.

TRIGONOMETRIC INEQUALITIES

By W. E. BYRNE
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Lexington, Va.

In differential calculus it is sometimes important to know whether simple functions involving trigonometric expressions are positive or negative. Such a case would arise in tracing the graph of

$$(1) \qquad \rho^2 = \sin \tfrac{1}{3}\,\Theta$$

All too frequently the student is mislead by methods in vogue in trigonometry and analytic geometry of substituting values of Θ at random to obtain values of ρ without any attempt at the precise determination of the critical values and then drawing in a smooth curve through the corresponding points.

The discussion will be carried out for the particular problem in question to show a more scientific method, which can be carried over into more complicated problems.

Only those values of Θ for which $\sin\frac{1}{3}\Theta \nless 0$ are of interest here since ρ, being real, has its square either 0 or positive. Furthermore it is known that a curve whose equation is given in polar coordinates will be closed if ρ is continuous and real between two consecutive Θ values for which ρ becomes 0 of ir ρ has a period which is an integral multiple of 2Π.

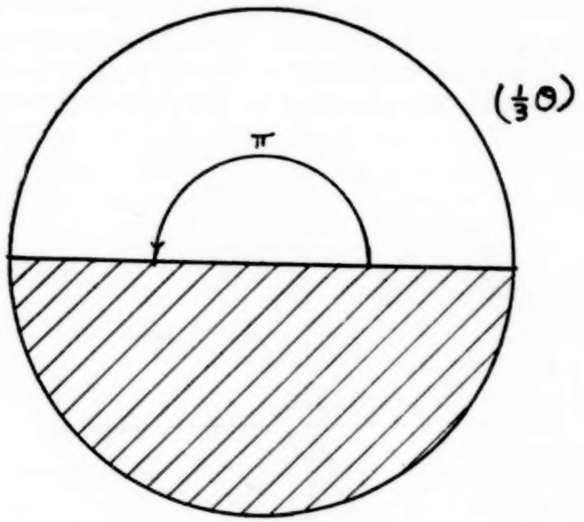
$\rho = 0$ if $\frac{1}{3}\Theta = k\Pi$, i. e. if $\Theta = 3k\Pi$, where k is an integer or 0. Furthermore $\sin\frac{1}{3}\Theta$ has a period 6Π. Hence the greatest interval necessary for consideration is of length 6Π, so $0 \ngtr \Theta < 6\Pi$ may be taken.

Since $\sin\frac{1}{3}\Theta$ is a continuous function of Θ, the values of Θ which

make $\sin\frac{1}{3}\theta = 0$ separate those for which $\sin\frac{1}{3}\theta > 0$ from those for which $\sin\frac{1}{3}\theta < 0$. Hence

$$0 < \theta/3 < \Pi \qquad \text{or} \qquad 0 < \theta < 3\Pi$$

gives the 0 interval required. This is intuitive if we represent what happens on the $\frac{1}{3}\theta$ circle diagram, indicating the "forbidden" values of θ by the hachured region.



$\sin\frac{1}{3}\theta > 0$

Figure 1.

Consider one determination

$$\rho = +\sqrt{\sin\tfrac{1}{3}\theta} \tag{2}$$

and trace the corresponding curve in the 0-interval $(0,3\Pi)$, with ρ maximum at $\theta = 3\Pi/2$. The other branch

$$\rho = -\sqrt{\sin\tfrac{1}{3}\theta} \tag{3}$$

may be attained by symmetry with respect to the pole. The drawing of the figure is left to the reader.

A slightly more complicated problem is to determine all values of θ for which

$$f(\theta) \equiv \cos\theta\sin3\theta > 0 \tag{4}$$

The following are the logical possibilities (some of which might prove to be vacuous):

Case 1. $\cos\theta > 0$, $\sin 3\theta > 0$

Case 2. $\cos\theta < 0$, $\sin 3\theta < 0$

The reader is asked to make the figures analogous to Figure 1 for each of the two parts in each case, then to superimpose the figures for case 1, to determine values of θ satisfying that case, and to carry out the same operation for case 2. The combined results are indicated in Figure 2.

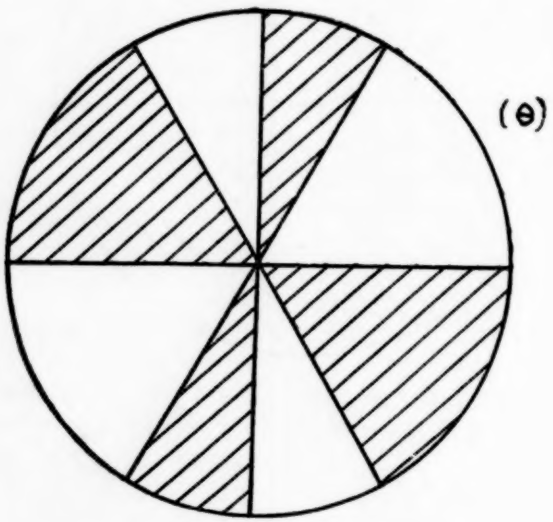


Figure 2

Analytically they may be expressed as follows:

$$2k\pi < \theta < 2k\pi + \pi/3$$

$$2k\pi + \pi/2 < \theta < 2k\pi + 2\pi/3$$

$$2k\pi + \pi < \theta < 2k\pi + 4\pi/3$$

$$2k\pi + 3\pi/2 < \theta < 2k\pi + 5\pi/3$$

The same result could have been obtained more readily by marking on a circle the terminal sides of θ corresponding to the roots of $f(\theta)=0$, following values of $f(\theta)$ by continuity, noting that (in this case) $f(\theta)$ changes sign each time it becomes 0, θ increasing, for instance.

The advantage of such a method is that it permits the rapid reduction of more or less complicated problems to simple ones by very intuitive processes not beyond the ability of high school students. Although such methods are not included in the average text, there is no reason why they could not be developed in trigonometry classes in spite of the statement made in the preface of a recent college textbook that "the content and method of elementary college mathematics has been standardized for such a long time that there is little room for originality and for deviation from many sources in the subject."

AN APPLICATION OF THE METHOD OF UNDETERMINED COEFFICIENTS TO SYMMETRIC FUNCTIONS

By H. L. SMITH
Louisiana State University

It is well known that every symmetric polynominal in n variables can be expressed uniquely as a polynomial in the n so-called elementary symmetric functions of those variables. The work of securing such an expression for a symmetric function is almost always carried out by means of certain direct methods of computation. It is the object of this note to show that the method of undetermined coefficients can be used for this purpose.

1. **Notation.** We shall denote by $T(m_1, \ldots, m_n)$ the term in $x_1, \ldots, x_n$ whose coefficient is 1 and in which $x_1, \ldots, x_n$ have respectively the degrees $m_1, \ldots, m_n$. We shall also denote by $S(m_1, \ldots, m_n)$ the simplest symmetric function of which $T(m_1, \ldots, m_n)$ is a term, that is, the sum of all formally distinct terms which can be obtained from $T(m_1, \ldots, m_n)$ by permutations of $x_1, \ldots, x_n$. The elementary symmetric functions of $x_1, \ldots, x_n$ are then

$$E_1=S(1, 0, \ldots, 0),\ E_2=S(1,1,0,\ldots,0),\ \ldots\ldots,\ E_n=S(1,1,\ldots,1).$$

We finally denote by $E(m_1, \ldots, m_n)$ the term in $E_1, \ldots, E_n$ with unit coefficient in which $E_1, \ldots, E_n$ are of degrees $m_1, \ldots, m_n$, respectively.

2. **The fundamental existence theorem.** The fundamental existence theorem referred to above states that $S(m_1, \dots, m_n)$ can be expressed uniquely as a linear combination of terms of the type $E(p_1, \dots, p_n)$. An excellent proof of this is to be found in Dickson, First Course in the Theory of Equations, p 130. This proof of Dickson's brings out certain important facts about this expansion. In the first place it shows that if $kE(p_1, \dots, p_n)$ is a term in the expansion of $S(m_1, \dots, m_n)$ then the degree of $k\ E(p_1, \dots, p_n)$ as a function of $x_1, \dots, x_n$ must be the same as that of $S(m_1, \dots, m_n)$. That gives

$$p_1+2p+3p+ \dots +np_n=m_1+m_2+ \dots +m_n \tag{1}$$

But this is not the only important fact that we observe. To state another one we need a concept on which the whole of Dickson's proof rests. This is as follows: A term $T(m_1, \dots, m_n)$ is *higher* than a term $T(r_1, \dots, r_n)$ if the first one of the numbers

$$m_1-r_1\ ,\ m_2-r_2\ ,\ \dots\dots ,m_n-r_n.$$

which is not zero, is positive. Clearly every $S(m_1, \dots, m_n)$ has a highest term, which will be $T(m_1, \dots, m_n)$ if $m_1 \nless m_2 \nless \dots\dots \nless m_n$. A check of Dickson's proof now leads to the conclusion that if $kE(p_1 \dots, p_n)$ is a term of the expansion of $S(m_1, \dots, m_n)$ then the highest term of $E(p_1, \dots, p_n)$ when expressed in terms of $x_1, \dots, x_n$ is not higher than the highest term of $S(m_1, \dots, m_n)$. Since the highest term of $E(p_1, \dots, p_n)$ is $T(p_1+p_2+ \dots +p_n, p_2+ \dots +p_n, \dots, p_n)$ it follows that if $m_1 \nless m_2 \nless \dots \nless m_n$ then the first number (if any) in the set

$$m_1-(p_1+ \dots +p_n), m_2-(p_2+ \dots +p_n), m_3-(p_3+ \dots +p_n), \dots, m_n-p_n, \tag{2}$$

which is not zero, is positive.

It is easily seen that $kE(m_1-m_2, m_2-m_3, \dots\dots, m_n)$ satisfies both of the requirements laid down. Reference to Dickson's proof shows that it is indeed a term in the required expression and that in it $k=1$.

3. **Rule for expansion of symmetric functions in terms of elementary ones.** We are thus led to the following:

Rule for expressing $S(m_1, \dots, m_n)$ $(m_1 \nless m_2 \nless \dots \nless m_n)$ *as a linear combination of terms of the form* $E(p_1, \dots, p_n)$.

(A) Determine all ordered sets $(p_1, \dots, p_n)$ of non-negative integers $p_1, \dots, p_n$ such that (1) is satisfied.

(B) Delete all such sets $(p_1 \, .. p_n)$ which do not satisfy the condition that the first number (if any) of the set (2) which is not zero shall be positive.

(C) Write down the equation

$$S(m_1, \ldots, m_n) = E(m_1 - m_2, m_2 - m_3, \ldots, m_n) + AE(p_1, \ldots, p_n)$$
$$+ BE(q_1, \ldots, q_n) + CE(r_1, \ldots, r_n) + \ldots, \qquad (3)$$

where $(p_1, \ldots, p_n)$, $(q_1, \ldots, q_n)$, $(r_1, \ldots, r_n)$, ...denote the various sets (other than $(m_1 - n_2, m_2 - n_3, \ldots, m_n)$) obtained in (B).

(D) Into (3) substitute numerical values of $(x_1, \ldots, x_n)$ and continue until enough equations are found to determine A,B,C,....

(E) Solve the resulting equations for A,B,C,...

4. **An example.** Let it be required to express S(4,2,1) as a linear combination of terms of the form $E(p_1 p_2 p_3)$.

(A) The equation

$$p_1 + 2p_2 + 3p_3 = 7$$

has the solution in non-negative integers given in the following table:

1, . . .	(7,0,0)
2, . . .	(5,1,0)
3, . . .	(4,0,1)
4, . . .	(3,2,0)
5, . . .	(2,1,1)
6, . . .	(1,3,0)
7, . . .	(1,0,2)
8, . . .	(0,2,1)

(B) The values of $m_1 - (p_1 + p_2 + p_3), m_2 - (p_2 + p_3), m_3 - p_3$ corresponding to the above values of $(p_1 p_2 p_3)$ are given by the following table:

1.	(−3,2,1)
2.	(−2,1,1)
3.	(−1,1,0)
4.	(−1,0,1)
5.	(0,0,0)
6.	(0,−1,1)
7.	(1,0,−1)
8.	(1,0,0)

Of these all must be deleted except the 5th, 7th, and 8th.

(C) Set

$$S(4,2,1)=E(2,1,1)+AE(1,0,2)+BE(0,2,1), \tag{4}$$

where A,B are to be determined.

(D) Set $x_1=x_2=x_3=1$ in (4) getting

$$6=27+3A+9B$$

or

$$A+3B=-7 \tag{5}$$

Next set $x_1=x_2=1,\ x_3=-1$ in (4) getting

$$2=1+A-B$$

or

$$A-B=1 \tag{6}$$

(E) From (5),(6)

$$A=-1,\ B=-2$$

so that

$$S(4,2,1)=E(2,1,1)-E(1,0,2)-2E(0,2,1)$$

or in ordinary notation

$$Sx_1^4x_2^2x_3=E_1^2E_2E_3-E_1E_3^2-2E_2^2E_3$$

THE EFFECT OF A CERTAIN WEEKLY SCHEDULE AT THE RATE OF ONE-HALF OF ONE PER CENT PER WEEK

By MRS. E. S. SAMUELS
Baton Rouge, La.

A request for the solution of the following problem has been received by the Department of Mathematics of Louisiana State University, and, since it is a typical problem of the business world, perhaps its solution would interest others:

Problem: *A financial company loans one hundred dollars returnable with interest at a flat rate of fifty cents per week for three hundred and twelve weeks (six years). Assuming that all money received is again reloaned on this same basis; what will be the total volume of loans made by the company during the first six years of operation?*

Solution: Notice that the ratio of the weekly payments to the loan is $^1/_{200}$ or .005 of itself; further, except for the original loan of one hundred dollars, the amounts loaned out during successive weeks increase by .005 of themselves respectively. Thus the weekly loans constitute a geometric series whose first term is .50, whose ratio is 1.005, and whose number of terms is 312. Therefore the amount loaned during the 312 weeks is $.50+05(1.005)+.50(1.005)^2+.05(1.005)^3+\ldots+.50(1.005)^{311}$ or $.50(1+1.005+1.005^2++.005^3+\ldots+1.005^{311})$. Using the formula for finding the sum of a geometric series, the above series reduces to $.50(1.005^{312}-1)/.005$ or \$374.049, the amount loaned during the six-year period in question.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
University, Miss.

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Miss.

Solutions

No. 22. Proposed by Everett E. Cook, University, Miss.:

A man died on a birthday before 1930 and his age then was $^1/_{29}$ of the year of his birth. How old was he on his birthday in 1900?

I. Solved by Mannie Charash, 1835 83rd Street, Brooklyn, N. Y.

Let his date of death be $1900+y$ $(y<30)$ and his date of birth be $1800+x$.

Then, $$100 - x + y = \frac{1800 + x}{29}$$

or $$y = \frac{30x - 1100}{29}$$

Solving as a congruence,

$$30x \equiv 1100, \text{ mod. } 29$$
$$x \equiv 27, \text{ mod. } 29$$

which leads to the only solution possible: $x = 56$ $y = 20$. Date of birth 1856 and age in $1900 = 44$.

II. Solved by Lucille G. Meyer, 3710 General Pershing, New Orleans, La.

Let $x =$ year of birth

Then $$1900 < x + {}^1/_{29}{}^x < 1930$$
$$1836\tfrac{2}{3} < x < 1865\tfrac{2}{3}$$
$$1836 < x < 1866$$

We seek integral values of x, satisfying this inequality which are multiples of 29 and find only one namely 1856. Therefore, in 1900 the man was 44 years old.

III. Solved by James F. Constantine, Child Welfare Research Station, University of Iowa, Iowa City, Iowa.

Let $x =$ year of birth

$y =$ year of death

$$x = \frac{29}{30} y$$

$$1900 < y < 1930$$

We seek integral values of y which are multiples of 30 and satisfying this inequality and find only one namely 1920. $x = 1856$ and he was 44 years old in 1900.

No. 18. Proposed by T. A. Bickerstaff, University, Miss.:

Eliminate θ from the system

$$\begin{cases} x = \cos 2\theta + 2\cos\theta \\ y = \sin 2\theta - 2\sin\theta \end{cases}$$

Solution by H. T. R. Aude, Colgate University, Hamilton, N. Y.

The given equations represent the hypocycloid of three cusps formed by a circle of radius 1 rolling on the inside of the circle $x^2 + y^2 = 9$, where the angle θ is measured clockwise from the axis OX to the line of centers of the two circles. (See N. C. Riggs' Analytic Geometry p. 135). The curve is completely traced as θ changes from 0 to 2π. The first arch is generated while θ changes from 0 to $2\pi/3$, x decreases from 3 to $-3/2$, and y decreases from 0 to $-3\sqrt{3}/2$. For the second arch θ changes from $2\pi/3$ to $4\pi/3$, x increases from $-3/2$ to -1 and then decreases to $-3/2$ while y increases from $-3\sqrt{3}/2$ to $3\sqrt{3}/2$. For the third arch θ changes from $4\pi/3$ to 2π, x increases from $-3/2$ to 3, and y decreases from $3\sqrt{3/2}$ to 0.

To eliminate θ from the two equations write the first one

$$2\cos^2\theta + 2\cos\theta - x - 1 = 0$$

and solve for $\cos\theta$. This gives

$$\cos\theta = (-1 \pm p)/2, \text{ where } p = \sqrt{3 - 2x}. \qquad (1)$$

The second equation can be written

$$2\sin\theta(\cos\theta - 1) = y.$$

Whence replacing $\cos\theta$ by its value in (1) and solving for $\sin\theta$, there results

$$\sin\theta = y/(-3 \pm p). \qquad (2)$$

It is seen upon examination that when the negative sign is taken in (1) and (2) the conditions apply to the second arch, while, with the positive sign, these equations apply to the first and third arch of the hypocycloid.

Squaring and adding equations (1) and (2) yields

$$\frac{(-1 \pm p)^2}{4} + \frac{y^2}{(-3 \pm p)^2} = 1.$$

Simplifying and substituting for p its value given in (1) gives

$$x^2+y^2+12x+9 = \pm 2(3+2x)^{3/2}, \qquad (3)$$

When the minus sign is taken equation (3) belongs to the second arch. Upon squaring and rearranging of terms there results the equation

$$(x-3)^3(x+1)+2(x^2+12x+9)y^2+y^4=0$$

which represents the complete hypocycloid, and it is the equation desired.

No. 19. Proposed by William E. Bryne, Virginia Military Institute, Lexington, Va.

The function

$$f(\theta)=\sin 3\theta/\cos\theta$$

is 0 for $\theta=0$ and for $\theta=\Pi/3$. In the open interval $(0, \Pi/3)$ $f(\theta)>0$, and since it is continuous it must have at least one maximum. Show that there is only one maximum and find its value in its simplest form.
Solution by H. T. R. Aude, Colgate University, Hamilton, N. Y.

Since

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

the function $f(\theta)$ to be examined for a maximum can be written

$$f(\theta)=\sin 3\theta/\cos\theta=\tan\theta \,.\, (3-4\sin^2\theta). \qquad (1)$$

Differentiating gives

$$f'(\theta)=(3-12\sin^2\theta+8\sin^4\theta)\sec^2\theta. \qquad (2)$$

For the first derivative to be zero it is necessary that

$$3-12\sin^2\theta+8\sin^4\theta=0$$

and this is true when

$$3-4\sin^2\theta=\sqrt{3} \tag{3}$$

It is readily seen that only the positive sign can be taken with $\sqrt{3}$. And since $\sin\theta$ is positive on the interval, it follows that

$$\sin\theta=\tfrac{1}{2}\sqrt{3-\sqrt{3}}. \tag{4}$$

Since there is only one value of θ between 0 and $\Pi/3$ for which the first derivative vanishes, there is only one maximum.

To find this maximum value, use the value of $\sin\theta$ given in (4) and find

$$\tan\theta=(3\sqrt{2}-\sqrt{6})/2 \tag{5}$$

Substituting the values given in (5) and (3) in (1) yields the maximum value of $f(\theta)$

$$f(\theta)=\sqrt{3}(3\sqrt{2}-\sqrt{6})/2=3(\sqrt{6}-\sqrt{2})/2=6\sin 15°.$$

No. 21. Proposed by T. A. Bickerstaff, University, Miss.:

The angles of a plane triangle form a geometric progression whose common ratio is 3. Show that the ratio of the perimeter to the smallest side is

$$8\sin\frac{10\Pi}{26}\sin\frac{11\Pi}{26}\sin\frac{12\Pi}{26}.$$

Solution by H. T. R. Aude, Colgate University, Hamilton, N. Y.

Since the angles : A, B, C of the triangle are in geometric progression with common ratio 3, it follows that

$$A=\Pi/13,\ B=3\Pi/13,\ C=9\Pi/13.$$

From the law of sines it follows that the ratio T, of the perimeter to the smallest side, is

$$T=\frac{a+b+c}{a}=\frac{\sin A+\sin B+\sin C}{\sin A}$$

$$= \frac{\sin \Pi/13 + \sin 3\Pi/13 + \sin 9\Pi/13}{\sin \Pi/13}$$

Since

$$\sin 9\Pi/13 + \sin \Pi/13 = 2\sin(5\Pi/13)\cos(4\Pi/13),$$

and

$$\sin 3\Pi/13 = \sin 10\Pi/13 = 2\sin(5\Pi/13)\cos(5\Pi/13),$$

it follows that

$$T = (2\sin 5\Theta/13)\cos(4\Pi/13) + 2\sin(5\Pi/13)\cos(5\Pi/13)/\sin \Pi/13$$

$$= \frac{2\sin 5\Pi/13}{\sin \Pi/13}(\cos 4\Pi/13 + \cos 5\Pi/13)$$

$$= \frac{2\sin 5\Pi/13}{\sin \Pi/13}(2\cos 9\Pi/26 \cos \Pi/26).$$

But

$$\cos \Pi/26 = \sin 12\Pi/26, \text{ and } \cos 9\Pi/27 = \sin 2\Pi/13 = 2\sin(\Pi/13)\cos(\Pi/13)$$

$$= 2\sin(\Pi/13)\sin 11\Pi/26.$$

Substituting for the two cosines these values and simplifying gives the desired result.

$$T = 8\sin 10\Pi/26 \cdot \sin 11\Pi/26 \cdot \sin 12\Pi/26.$$

Problems for Solution

No. 23. Proposed by William E. Bryne, Virginia Military Institute, Lexington, Va.

Find the most general solution of the simultaneous equations,

$$\cos\Phi = \frac{1+\cos\alpha}{2\left|\cos\dfrac{\alpha}{2}\right|}$$

$$\sin\Phi = \frac{\sin\alpha}{2\left|\cos\dfrac{\alpha}{2}\right|} \quad \text{where}$$

$|\;|$ indicates absolute value and α is a constant, $\cos\frac{\alpha}{2} \neq 0$.

No. 24. Proposed by William E. Bryne, Virginia Military Institute, Lexington, Va.

Prove that if k is a given positive integer, the inequalities

$$\frac{(s-1)(s-2)}{2} < k \ngtr \frac{s(s-1)}{2}$$

Admit one and only one positive integral solution s. Find this solution for $k=1000$.

No. 25. Proposed by E. C. Kennedy, College of Mines, El Paso, Tex.

Solve for least positive value of x to within an error of about one second or less. The solution may be obtained very easily if the expression is simplified properly.

$$\sqrt{\sin 2x} = \sqrt{2}\cos x[\sqrt[u]{4.76}-1]$$

Where $u = 1+\sqrt{\tan x}$

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